

# POINT – A TOOL TO OPTIMIZE INTERPLANETARY TRAJECTORIES

Sven Schäff<sup>(1)</sup>, Martin Jürgens<sup>(2)</sup>, Andreas Wiegand<sup>(3)</sup>

<sup>(1)</sup> Astos Solutions GmbH, Meitnerstrasse 10, 70563 Stuttgart, Germany, Email: sven.schaeff@astos.de

<sup>(2)</sup> Astos Solutions GmbH, Meitnerstrasse 10, 70563 Stuttgart, Germany, Email: martin.juergens@astos.de

<sup>(3)</sup> Astos Solutions GmbH, Grund 1, 78089 Unterkirnach, Germany, Email: andreas.wiegand@astos.de

## ABSTRACT

The Program to Optimize INterplanetary Trajectories (POINT) is presented in this paper. The software tool supports both impulsive as well as low thrust manoeuvres and offers different optimization techniques. Available are global and local methods. The tool offers several techniques to optimize interplanetary trajectories like the patched conics approach, a multiconics method and the sinusoids as a shape approach. POINT is an event driven tool which can be also used for launch window analyses. Different models for environment and spacecraft components are available and it's plotting features for 2D and 3D as well detailed reports complete the tool's capabilities.

## 1 SOFTWARE OVERVIEW

Cornerstone or flagship missions to the inner and outer planets as well as deep space missions are always of high scientific interest. But especially the small bodies of our solar system like asteroids and comets attract more and more interest not only for exploration and exploitation but also for mitigation, as some of them pose a certain risk for collision with Earth. In principle, spacecrafts can use high thrust chemical propulsion or low thrust electric propulsion which is fed by solar or nuclear energy. In addition, gravity assist manoeuvres are used to reduce fuel consumption. But they add complexity to the mission as launch windows are typically very short.

This leads to a wide variety of possible interplanetary missions. For mission planners it is essential to have a flexible, fast and reliable tool which deals with all these kind of missions to compute optimal transfer trajectories.

POINT is intended to support the analysis and design of interplanetary trajectories. It can be used in feasibility studies for the fast assessment of typical quantities coming from mission analysis like total  $\Delta v$  requirements, time of flight, departure and arrival conditions. The software tool consists of two basic components: a trajectory propagation tool, also known as the POINT model, and an optimization tool. The latter one is the Graphical Environment for Simulation and Optimization (GESOP) and is augmented with the propagation tool (described later in this section).

GESOP [1] is a software tool for almost all kind of optimization problems. It is mainly used for aerospace

applications (astronautics and aviation), but also extensively in automotive, robotics, medical research and bio-engineering. GESOP is a software system for analysis of multi-phase dynamic systems governed by nonlinear differential equations, constraints, and cost functions. Constraints can be initial/final boundary conditions and path constraints in single or multi phase(s), or any combination of them. Each constraint is defined either as inequality or as equality. The pattern of the cost functional as objective is very similar: It consists of initial/final cost terms and integrated cost functions. The basic scheme of the software is illustrated in Figure 1, where the upper part depicts the interface with the POINT model.

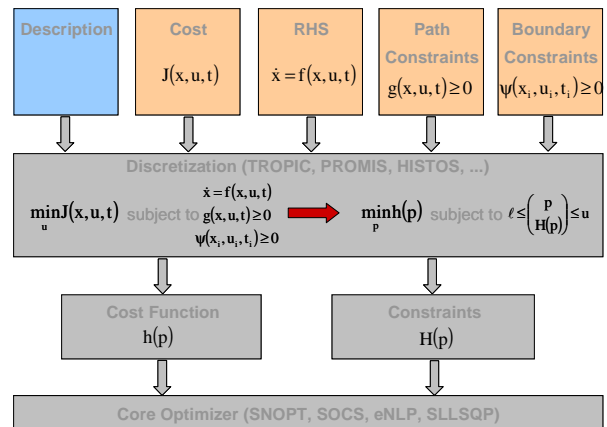


Figure 1. Basic scheme of GESOP interfaces between user model and optimizer.

The GESOP software includes different optimization techniques:

- Direct collocation methods (e.g. SOCS [2], eNLP).
- Multiple shooting method PROMIS.
- Hybrid method CAMTOS (selection of collocation and shooting method in each phase).
- Evolutionary algorithm CGA.

The software combines the optimizers with an initial guess generator, a simulation program, and several evaluation tools (Parameter & Grid Inspector; Tools Inspector; optimization monitor GISMO; ...) plus a plotting tool for 2D and 3D.

The propagation model is intended to provide the

optimization tool with all necessary data through interfaces. Therefore it consists of several parts, such as environment modelling, computation of constraints and cost terms, as well as the propagator. These parts are described in the sections 3 to 6.

## 2 OPTIMIZATION

The optimization mode optimizes the defined objective function subject to several (optional) constraints. In general two types of optimizers are available for POINT: a genetic algorithm and an SQP method. The genetic algorithm can be used to find the globally optimal interplanetary transfer or as initial guess for the gradient based search method, where the SQP method is used to refine the trajectory.

For global optimization the Simple Genetic Algorithm has been implemented and is introduced in the next section 2.1. For the local optimization two highly sophisticated gradient based algorithms are integrated. An overview is given in section 2.2.

### 2.1 Simple Genetic Algorithm

The Simple Genetic Algorithm (SGA) is intended for the optimization of a patched conics model. The basic structure follows:

1. Initialize a random population of individuals.
2. Evaluate the fitness of all individuals.
3. Select a sub-population for offspring reproduction.
4. Recombine the selected parents.
5. Perturb the mated population randomly.
6. Evaluate the new fitness.
7. Go to step 3.

#### Selection Schemes

As selection schemes are available:

- Tournament selection,
- Roulette wheel selection,
- Remainder stochastic independent sampling, and
- Stochastic universal sampling.

The tournament selection randomly selects two individuals and copies the best one into the next generation. For roulette wheel selection a biased roulette wheel is created where each individual receives a roulette wheel slot proportional to its fitness. The remainder stochastic independent sampling allocates samples according to the integer part of the expected value. The fourth method uses a single wheel spin constructed as described above but spun with a number of equally spaced markers equal to the population size.

#### Crossover Operators

One of the following crossover operators can be selected:

- Arithmetic crossover,
- Heuristic crossover, and
- Box crossover.

Arithmetic crossover produces two complimentary linear combinations of the parents. Heuristic crossover uses values of the fitness function in determining the direction of search and generates a single offspring from two parents. Box crossover is similar to arithmetic crossover: The two parents define the corners of a box and the offspring is generated within this box.

#### Mutation Operators

Available mutation operators are:

- Uniform mutation,
- Non-uniform mutation, and
- Boundary mutation.

The uniform mutation randomly selects a variable and sets it equal to a uniform random number, whereas the non-uniform mutation randomly selects a variable and mutates it. The boundary mutation randomly selects a variable and sets it equal to either its lower or upper bound.

#### Constraints

Some constraints are automatically created by the model (e.g. minimum flyby altitude) and others are from the user (e.g. maximum total delta-v). In fact, there is no difference between automatic and user defined constraints. The genetic algorithm considers constraints by replacing the fitness function with an augmented Lagrangian function. All inequality constraints are transformed to equality constraints and a generalized augmented Lagrangian function is formulated.

#### Objectives

The objective function is usually defined by the user. Several objectives can be formulated but proper scaling of them is essential.

#### Hybridization

An advanced feature is the possibility to hybridize the SGA with a local search method. It is a self-adaptive hybrid method where the number of local search iterations and the local search probability has to be specified by the user. The number of local search iterations defines how long the local search lasts before switching back to the global genetic algorithm search.

### 2.2 Local Optimization Methods

To refine the solution found by the genetic algorithm two local optimization methods are provided as standalone optimizers: SOCS [2] and SNOPT [3]. Furthermore SNOPT is used for the local search of the

Simple Genetic Algorithm introduced in section 2.1. Both are gradient based algorithms using a Sequential Quadratic Programming (SQP) algorithm that obtains search directions from a sequence of quadratic programming sub-problems. Each quadratic programming sub-problem minimizes a quadratic model of a certain Lagrangian function subject to a linearization of the constraints. An augmented Lagrangian merit function is reduced along each search direction to ensure convergence from any starting point. Both methods require the derivatives of the objective function and of the constraints with respect to the optimization variables.

The advantage of these methods is that they can handle even high and very high parameterized optimization problems. SNOPT solves problems up to 10000 parameters whereas SOCS solves problems with more than several 100000 parameters. Very sophisticated low thrust transfer models can be set up with e.g. vector control, where thrust magnitude and direction are represented by a Cartesian vector.

### 3 ENVIRONMENT

The sophistication of the environment model not only depends on the used propagator. For example, the type of the mission (chemical or electrical propulsion) and the computational speed affect the level of modelling, too.

Always defined are the celestial objects which serve as destinations of the mission. The objects can be either the planets of the solar system, their moons, or additional objects like asteroids and dwarf planets. Their ephemerides are provided via external libraries such as JPL DE405 or SPICE, or by a set of orbital elements plus epoch. An example of the definition of an celestial body is given in Figure 2.

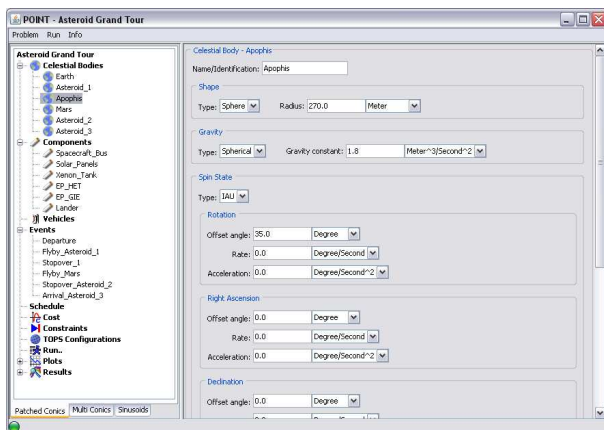


Figure 2. A celestial body is defined by shape, gravity, and spin state.

#### 3.1 Perturbing Accelerations

Supported are perturbing accelerations which are caused by either:

- Third bodies (according to Battin [4]),
- Central body oblateness,
- Solar radiation pressure, or
- Low thrust propulsion.

#### 3.2 Three Body Problem

The restricted three body problem is used as model. The three bodies are the spacecraft, which is assumed to have a negligible mass so that it is affected by, but does not affect the motion of the other two bodies, and a celestial body and its parent body (being the central body). For example, if the celestial body is a planet like Jupiter the parent body is the Sun, in case it is Titan the parent would be Saturn. The accelerations of the spacecraft are computed with the gravitational constants of the celestial body and its central body, as well with the relative position vectors between the three objects.

### 4 SPACECRAFT COMPONENTS

A spacecraft typically consists of components like structure, power generators, tanks and propulsion. Sometimes also payloads and other components are attached to the vehicle. All these different kinds of spacecraft components must be modelled in proper way. Available components are:

- Structure,
- Payload,
- Solar Power Generator,
- Nuclear Power Generator,
- Electric Propulsion System,
- Chemical Propulsion System, and
- Tank.

Basic components such as structures and payloads have structural mass and (optionally) reflective properties for the computation of the solar radiation pressure. A tank has two masses: one each for structure and fuel. The power generators provide the spacecraft with the electrical energy. Two types are available: While the provided power of the nuclear power generator (e.g. nuclear reactor) stays constant with the time, the provided power level of the solar power generator depends on the distance to the Sun. The propulsion systems are basically identical. Such a component is specified with the thrust magnitude, the specific impulse and the mass flow rate. The electric propulsion system needs electrical power and the power consumption is needed as input to compute its properties (see Figure 3). Furthermore, each component may have a power consumption (e.g. for heating) which is considered for the performance of electric propulsion systems.

Once the basic components are defined the vehicle is set up with a certain amount of them, as for instance a vehicle may consist of a single spacecraft bus, a solar array, two payloads, and four engines attached to one tank. Not all components must be used as some may be

docked to the vehicle later during an event (e.g. sample container of a sample return mission). Other components can be jettisoned during an event. For example, a lander could be released at a celestial body or an empty tank is jettisoned after a deep space maneuver.

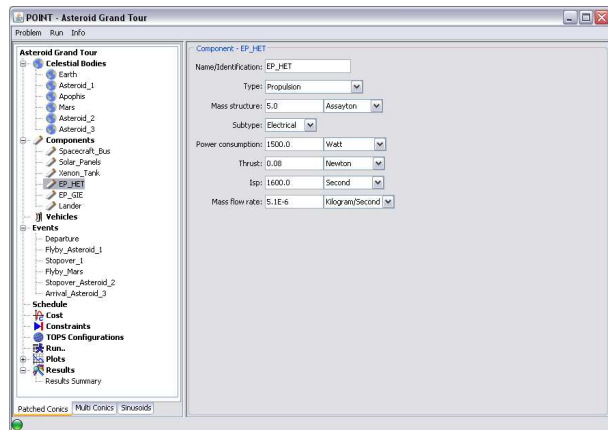


Figure 3. An electric propulsion system and its properties part of the spacecraft components.

## 5 EVENTS

POINT is event driven and a sequence of events must be defined like departure, swing-by, deep space manoeuvre, and arrival, which are executed in the simulation process. The tool allows for multiple swing-bys as well as stopovers and the whole mission definition is done via a schedule.

### 5.1 Departure and Escape

The departure or escape manoeuvre injects the spacecraft from an elliptical parking orbit to a hyperbolic escape orbit. The parking orbit is defined through the periapsis and apoapsis radius (or altitude).

### 5.2 Arrival and Capture

For an arrival or capture the vehicle is inserted from a hyperbolic trajectory into an orbit around the celestial body. The parking orbit is defined through the periapsis and apoapsis radius (or altitude).

### 5.3 Flyby

When a spacecraft approaches a celestial object it is perturbed by the mass of that object. This effect is known as flyby, swing-by, or gravity assist. The computation of the spacecraft's trajectory is approximated by considering two conics, one conic (incoming hyperbola) before closest approach to the object and the second after closest approach (outgoing hyperbola). The spacecraft velocity vector is rotated through the turning angle by the gravitational effect of the astronomical object whereas the magnitude of the

spacecraft velocity is not changed.

However, the velocity of the spacecraft with respect to the central body of the flyby object is changed. To obtain the velocity of the spacecraft with respect to the central body, the velocity of the planet with respect to it must be added to the spacecraft velocity vector. The velocity of the spacecraft with respect to the central body can increase or decrease, depending on the encounter geometry.

Gravity-assist manoeuvres are treated as instantaneous. The sphere of influence of a celestial body is assumed to have zero radius. Thus, the position vector is not affected where the incoming position vector is the outgoing position vector, which is the position vector of the celestial body. The deflection angle of the flyby is calculated from the incoming and outgoing relative velocities. The periapsis altitude is computed from the hyperbola. An automatic constraint is evaluated to fulfil the minimum periapsis altitude provided as input.

With the calculated periapsis radius the velocity in the periapsis can be determined for the incoming and outgoing orbit. All these entities yield the delta-v in the periapsis. In general a powered flyby is required since the incoming and outgoing relative velocities at the flyby object are not equal. In case of an unpowered gravity assist an automatic constraint is created ensuring the delta-v stays zero.

### 5.4 Stopover

A stopover is an event which actually consists of two events: arrival and departure. Such an event is appropriate for instance for a multi-rendezvous tour with a certain stay time for e.g. detailed scientific investigations. The stay time and its bounds must be given and an automatic constraint for it is created ensuring that it is fulfilled.

### 5.5 Deep Space Manoeuvre

A deep space manoeuvre is typically applied far beyond any celestial body, hence the name. Usually such a manoeuvre is used to change the trajectory to trigger a gravity assist or to correct the orbit path. It is an impulsive manoeuvre with instantaneous change of the spacecraft's velocity. The magnitude of the velocity change is the delta-v and must be supplied. It can be either constrained or become part of the objective function.

## 6 PROPAGATORS

Three types of propagators are available within POINT: a conic propagator, a multiconic propagator, and a sinusoids propagator. Propagator selection depends upon user needs, such as simple and fast simulations for parametric feasibility analysis, or more detailed modelling of the physics of the problem.

## 6.1 Patched Conic

A spacecraft is to be transferred from a given orbit around a celestial body to an orbit around another celestial object with eventual gravity assists or other events at a number of intermediate objects. The problem is formulated using a sequence of two-body problems beginning with the initial body, followed by its parent body, the required intermediate bodies, and finally the target object as central body.

In the patched conic approximation the spacecraft is influenced only by the gravitational field of a celestial body when it is within its sphere of influence, and it is influenced only by the gravity of the parent body (the central body of the celestial object) when it is outside any activity sphere except of the one of the parent's body. Thus, the spacecraft is always in a two-body orbit with respect to either the parent body or one of its objects. The trajectory is based on simple conics (ellipses and hyperbolas), which are "patched". As already mentioned, the interplanetary mission is divided into several parts: the initial phase with an initial state, several cruise phases with transfer ellipses between the celestial bodies coupled with events, and the final phase with final state.

The interplanetary transfer arcs between two planets are based on Lambert problem transfers. The Lambert solver computes the initial and final velocities given the initial and final position and transfer time. The Lambert problem is solved using universal variables as described in [5].

With the patched conic method no trajectory propagation is necessary. The trajectory is based on simple conics. Only the sequence of celestial bodies together with the events and the encounter times must be specified.

## 6.2 Multiconic

The principle of the multiconic propagator is that the trajectory is split into multiple conics for a given number of time steps. An optional variable stepsize algorithm is supported to compute the time steps. Each sub-conic is propagated using either the Keplerian two body model or the (optional) three body problem described in section 3.2. At each time step, the position of the spacecraft is calculated by two Keplerian propagations: one with respect to the body whose gravitational attraction is the strongest, typically being the central body; and a second propagation with respect to the second most important celestial object in sense of gravitational attraction. The same steps are repeated taking the previous final point as the initial point of the next iteration. At each time step, the main body and the secondary body must be determined. Other kinds of perturbations as described in section 3.1 can be added to the propagator making this method applicable for low thrust transfer.

The multiconic method offers good results for a chemical propelled spacecraft on a journey with several flybys as well as for low thrust propelled spacecrafts without a large increase in computation time. It is a quick and precise method, halfway between the patched-conic method and numerical integration. The computational time is halved with respect to numerical integration and almost all of the error made by the patched conics method is corrected.

The optimization problem consists of two stages. In the first stage the complete end-to-end trajectory of the full mission is generated. Then the trajectory is broken up into a sequence of transfer legs, where the boundary between two successive legs is referred to as a trajectory breakpoint. The control points for a trajectory leg are placed at the events. The placement of the trajectory breakpoints is arbitrary. If no breakpoints have been specified, the breakpoint will be placed at the midpoint between two control points. Breakpoints and control points occur in an alternating fashion along the complete trajectory.

The initial leg is generated by forward propagation from the first control point to the first trajectory breakpoint and the last leg is generated by backward propagation from the last control point to the last break point. In case of intermediate events the trajectory leg is the result of two trajectory propagations: first, a backward propagation from the control point to starting break point of the leg; second, a forward propagation from the control point to the ending break point of the leg. The trajectory generation process is complete when all trajectory legs have been determined in this way.

The legs are contiguous in time. Initially these legs will not be continuous in either position or velocity. The discontinuity is computed as offsets in position and velocity. Continuity in the final trajectory is achieved by imposing constraints on the optimization process.

The second stage of the optimization problem contains the optimization itself. The objective function is the sum of all defined objectives such as the propellant consumption.

## 6.3 Sinusoid

Not only for impulsive transfer but also for low thrust transfers a large number of mission scenarios needs to be evaluated. The search space can become very large as the launch and arrival windows are typically of several years. Including also a large range of mission design parameters such as for the propulsion system requires an efficient generation, evaluation, and analysis of a large number of trajectories. In order to make this step efficient for low thrust transfers Petropoulos [6] proposed to model trajectories by sinusoids.

This shape approach enables to find trajectories with reasonable thrust profiles. Main draw back is the planar model as out-of-plane components are neglected. But the out of plane components can be approximated.

Fortunately for most missions involving the solar system bodies the introduced error by this method stays small as these objects are typically found close to the ecliptic with only small inclinations.

The shape of the trajectory is modelled as exponential sinusoid in polar representation by a few parameters and a variety of realistic transfer legs can be obtained. Once the shape is determined the thrust profile is computed.

The sinusoids propagator is used with an evolutionary algorithm and shows good results for trajectory legs influenced by low thrust propulsion systems. It perfectly serves as initial guess for more detailed optimization (if required) with a gradient based method and classical Keplerian dynamics including highly sophisticated environment models.

## 7 PLOTTING & OUTPUT

POINT is able to run single trajectory simulations for specific missions or multiple simulations. Multiple trajectory simulations are used for parametric scans. For example, it is used to generate contour plots for launch window analysis, also known as pork-shop plots. For the single trajectory simulations all typical entities are provided as output functions and can be plotted either in 2D (e.g. radius versus flight time) or 3D.

An advanced feature of POINT is the available output as report. The result summary briefly summarized all important entities related to the optimization and simulation process such as objective function and the constraint violations. Additionally, the maximum and minimum values of all output functions can be given, for example the maximum distance to the Sun and its date/time. The problem summary shows all relevant mission parameters for the defined and/or optimized events, components etc.

## 8 CONCLUSIONS

Offering different optimization techniques and propagators, POINT is the ideal software tool for analysis of interplanetary missions. It can be used to set up missions ranging from grand-tours, sample returns to multiple rendezvous and escape trajectories. The tool can be used for impulsive and low thrust transfers or a mixture of them. The flexibility of the software allows the user to achieve fast and reliable results for initial phases of mission design as well as for detailed trajectory design in later project phases. The plotting capabilities as well as the detailed summaries and outputs provide the user with additional information.

## 9 REFERENCES

1. *GESOP User Manual Release 7.0*, Astos Solutions GmbH, Unterkirnach, Germany, 2010.
2. *SOCS User's Manual Release 7.1*, The Boeing Company, Seattle, Washington, 2009.

3. Gill, P.E., Murray, W., Saunders, M.A. (2002). *User's Guide for SNOPT Version 6*, Department of Mathematics, University of California, San Diego, California.
4. Battin, R.H. (1999). *An introduction to the Mathematics and Methods of Astrodynamics*, AIAA Education Series, Reston, Virginia.
5. Bate, R.R., Mueller, D.D., White, J.E. (1971). *Fundamentals of Astrodynamics*, Mineola, New York.
6. Petropoulos, A.E. (2001). *A Shape-Based Approach to Automated Low-Thrust Gravity-Assist Trajectory Design*, Purdue University, West Lafayette, Indiana.